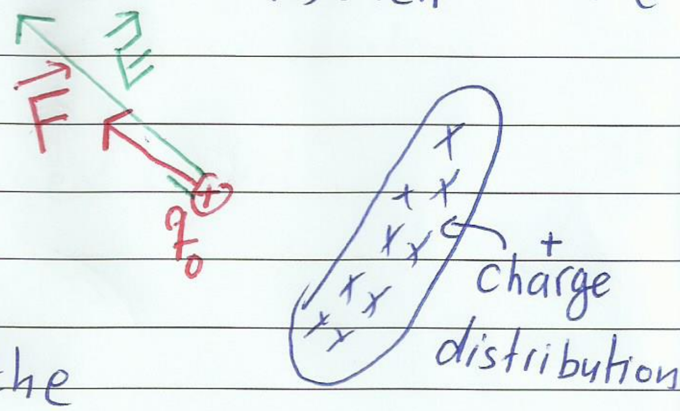
Chapter 22 - Electric Fields

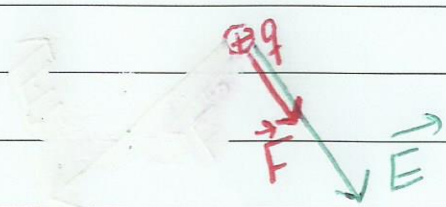
The Electric Field: $\vec{E}$

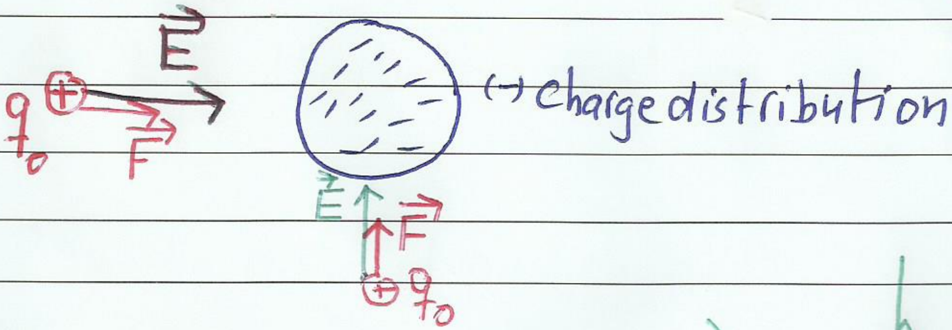
Is the electric Force from a charg distribution acting on a very small test charge $(+q_0)$ near the charge distribution, then divide the Force by q_0



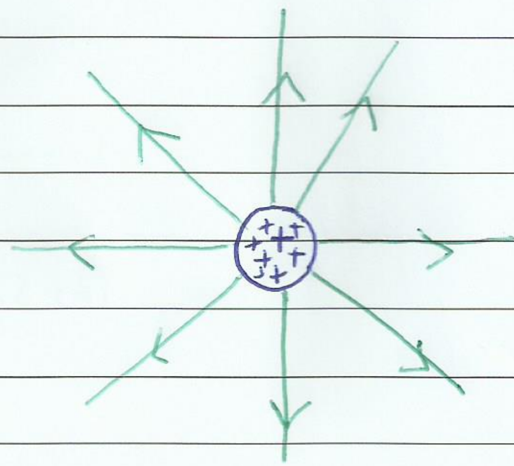
$$\vec{E} = \frac{\vec{F}}{q_0} \quad N/C$$

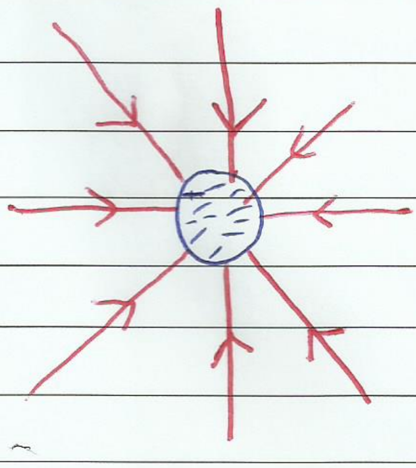
$\Rightarrow$ Electric Field is the electric Force acting on +1C

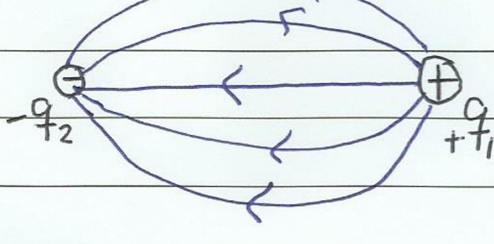




Electric Field lines:





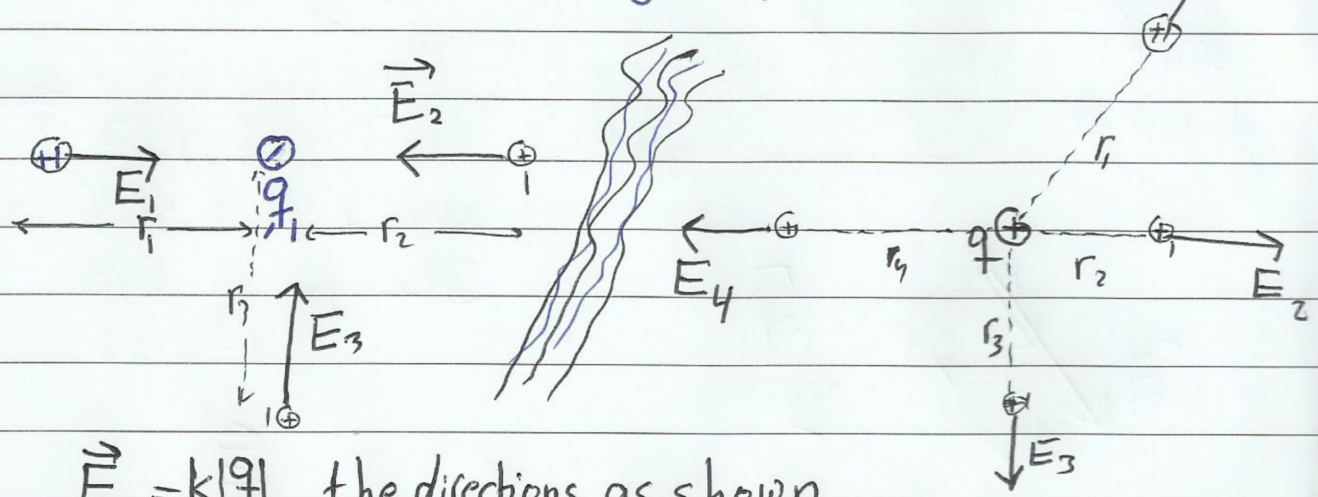


Electric Field lines extends away from positive charge and toward negative charge

In this Course we will find

$\vec{E}$
1) Due to a point charge
2) Due to a set of point Charges
3) Due to a Continous Chargedistribution

1] $\vec{E}$ Due to a point charged particles



$\vec{E} = k\frac{|q|}{r^2}$ the directions as shown

2] $\vec{E}$ Due to a Set of Point charges:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots\cdots$$

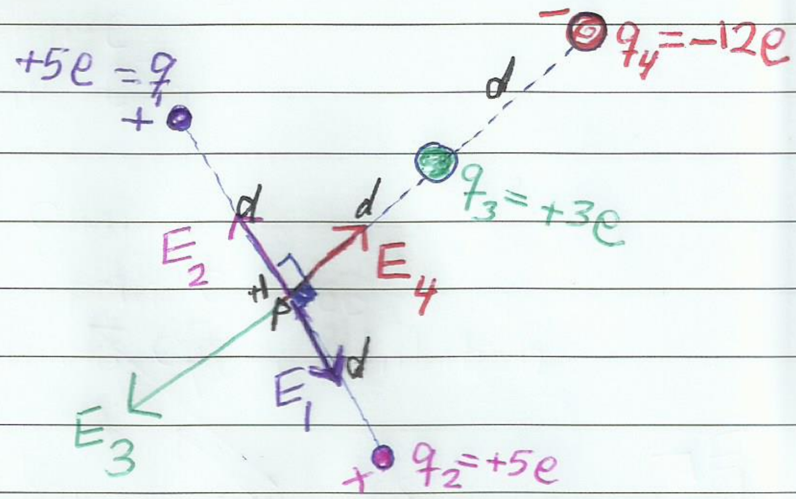
Solve sample Problem (22.01)

(22-32) I rotate the figure slightly

$d = 8\,\mu m$

find the $\vec{E}_{net}$ at point P?

$E = k\frac{q}{r^2}$



$E_1 = E_2 \Rightarrow \vec{E}_1 + \vec{E}_2 = 0$

$$E_3 = 9\times10^9 \times \frac{q_3}{d^2}$$

$$= \frac{9\times10^9 \times 3\times1.6\times10^{-19}}{(8\times10^{-6})^2} = \frac{4.32\times10^{-9}}{64\times10^{-12}}$$

$E_3 = 67.5\,N/C$ the direction shown

$$E_4 = 9\times10^9 \times \frac{q_4}{(2d)^2} = \frac{9\times10^9\times12\times1.6\times10^{-19}}{(16\times10^{-6})^2} = \frac{1.728\times10^{-8}}{(16\times10^{-6})^2}$$

$= 67.5\,N/C$ the direction as shown

$\vec{E}_3 + \vec{E}_4 = 0$

$\vec{E} = 0$

The Electric Field due to an Electric Dipole

$\vec{P} = q\vec{d}$

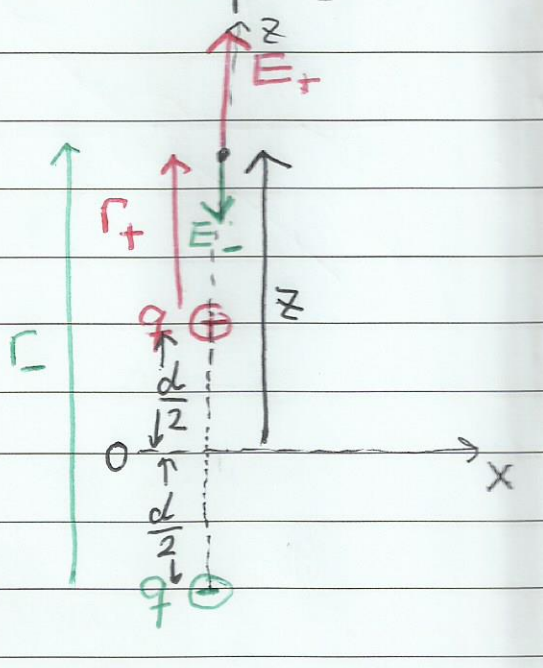
$\oplus q$

$d \uparrow \vec{P}$

$\ominus q$

$\vec{P}$ = Electric Dipole moment $= q\vec{d}$ Cm

Find the $\vec{E}$lectric Field at apoint along z-axis z distance from the origin?



$r_+ = z - \frac{d}{2} = z - a \quad , a = \frac{d}{2}$

$r_- = z + \frac{d}{2} = z + a \quad , a = \frac{d}{2}$

$$E_+ = \frac{kq}{r_+^2} = \frac{kq}{(z-a)^2} \quad \text{upward } +\hat{k}$$

$$E_- = \frac{kq}{r_-^2} = \frac{kq}{(z+a)^2} \quad \text{downward } -\hat{k}$$

$$\vec{E} = \frac{kq}{(z-a)^2} + \frac{-kq}{(z+a)^2} = \frac{kq[(z+a)^2-(z-a)^2]}{[(z-a)(z+a)]^2}$$

$$= \frac{kq[(z^2+a^2+2az)-(z^2+a^2-2az)]}{(z^2-a^2)^2}$$

$$= \frac{kq(4az)}{(z^2-a^2)^2}, \text{ remember } P = qd = q(2a)$$

$$\vec{E} = \frac{kP(2z)}{(z^2-a^2)^2} \quad \text{upward}$$

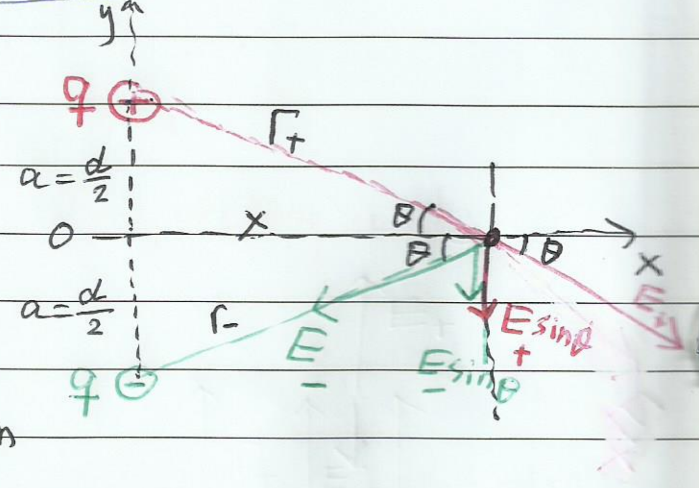
for $z \gg a \quad (z^2-a^2)^2 \longrightarrow z^4$

$$E = \frac{kP(2z)}{z^4} = \frac{2kP}{z^3} = \frac{P}{2\pi\epsilon_0 z^3} \quad \text{upward}$$

$$\vec{E} = \frac{P}{2\pi\epsilon_0 z^3}\hat{k}$$

(22-55) $P = qd = q(2a)$

Find $\vec{E}$ for the given an electric Dipole along the x-axis, a distance X From the origin?



$$E_+ = \frac{kq}{r_+^2} = \frac{kq}{(x^2+a^2)} \quad \text{as shown}$$

$$E_- = \frac{kq}{r_-^2} = \frac{kq}{(x^2+a^2)} \quad \text{as show}$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- \longrightarrow E_x = E_+\cos\theta + {}^-E_-\cos\theta = 0$$

$$\longrightarrow E_y = {}^-E_+\sin\theta + {}^-E_-\sin\theta = {}^-2E_+\sin\theta$$

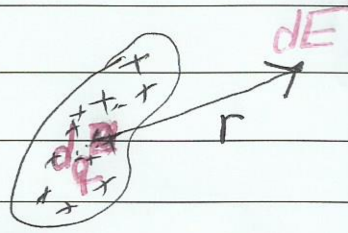
$$E_y = (-)2\left(\frac{kq}{(a^2+x^2)}\right)\left(\frac{a}{\sqrt{a^2+x^2}}\right) = \frac{(-)k(2aq)}{(x^2+a^2)^{3/2}}$$

For $d \ll x \Rightarrow (x^2+a^2)^{3/2} \longrightarrow (x^2+0)^{3/2} = x^3 \Rightarrow \vec{E} = \frac{P}{4\pi\epsilon_0 x^3}(-\hat{j})$

$\vec{E}$ due to a Continous Charge Distribution:

$$dE = k\frac{dq}{r^2}$$

$$E = \int dE$$

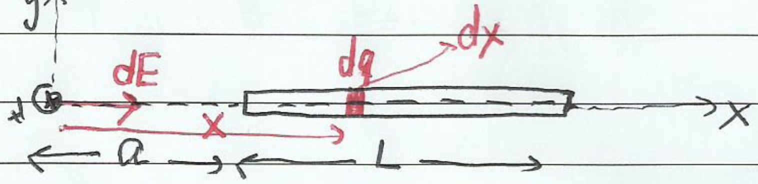


We will consider several Cases

Problem(22–13) Linear charge distribution

$\vec{E}$ due to a line of charge

Rod
- length = L = 8.15 cm
- $q = -4.23 fC$
- λ (linear charge density) $= \frac{q}{L}$



$$\lambda = \frac{-4.32\times10^{-15}}{8.15\times10^{-2}} = = -5.3\times10^{-14}\ C/m$$

b) Find $\vec{E}$ at a point along the x–axis a distance $a = 6.00\,cm$ from one end?

take you coordinate system at the point

dq is at a distance (x) from the point

$$dE = k\frac{dq}{x^2}$$, the width of dq is dx (very small)

$$dq = \lambda dx\ C$$

$$dE = k\frac{\lambda dx}{x^2}$$ $\Rightarrow$ the charge extends from $x_1 = a$ to $x_2 = L+a$

$$E = k\lambda\int_a^{L+a}\frac{dx}{x^2} = k\lambda\left[\frac{-1}{x}\right]_a^{L+a} = k\lambda\left[\frac{-1}{L+a} + \frac{1}{a}\right] = k\lambda\left[\frac{-a+L+a}{a(L+a)}\right]$$

$$E = \frac{\lambda L}{4\pi\epsilon_0 a(L+a)} = \frac{q}{4\pi\epsilon_0 (a)(L+a)}$$, $a = 6cm$, $L = 8.15cm$, $q = 4.23\times10^{-15}C$

$\vec{E} = 4.48\times10^{-3}\,N/C\ \hat{i}$

c) Find E at a point $a = 50m$? $\vec{E} = 1.5\times10^{-8}\,N/C\ \hat{i}$

d) Not that $a \gg L \Rightarrow a(L+a) \Rightarrow a(0+a) = a^2$

$E = \frac{kq}{a^2}$ as a Point charge $E = 1.5\times10^{-8}\,N/C\ \hat{i}$

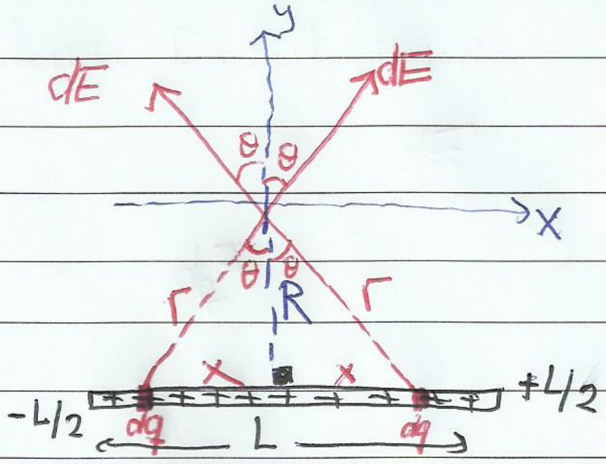
Problem (22-10)

A line of charge

rod → Length = L = 16 cm

rod → $q = 9.25\ pC = 9.25\times10^{-12}\ C$

rod → $\lambda = \frac{q}{L} = 5.78\times10^{-11}\ C/m$



Find $\vec{E}$ at a point a distanc $R = 6cm$ from the rod along its perpendicular bisector

$\cos\theta = \frac{R}{r}$

$r = \frac{R}{\cos\theta} = R\sec\theta = R\sec\theta$

take dq ~~to~~ be a distance (x) from the origin

$dE = \frac{k\,dq}{r^2}$, $dq = \lambda dx$,

$\tan\theta = \frac{x}{R}$

$x = R\tan\theta$

$dx = R\sec^2\theta\, d\theta$

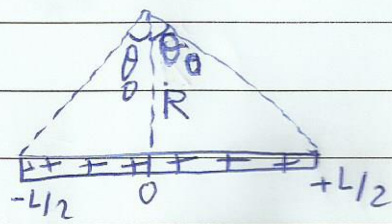
$dE_x = 0$ from symmetry

$E_x = 0$

$dx = R\sec^2\theta\, d\theta$

$(dE)_y = dE\cos\theta$

$$E_y = \int dE_{\cos\theta} = k\int \frac{dq_{\cos\theta}}{r^2} = k\int_{-L/2}^{+L/2} \frac{\lambda dx}{r^2}\cos\theta$$



$$E_y = k\lambda\int \frac{(R\sec^2\theta\, d\theta)}{R^2\sec^2\theta}\cdot\cos\theta = \frac{k\lambda}{R}\int_{-\theta_0}^{+\theta_0}\cos\theta\, d\theta$$

$\theta_0 = \tan^{-1}\left(\frac{L/2}{R}\right)$

$$= \frac{\lambda}{4\pi\epsilon_0 R}\left[\sin\theta\right]_{-\theta_0}^{\theta_0} = \frac{\lambda}{4\pi\epsilon_0 R}\left[\sin\theta_0 - \sin{-\theta_0}\right]$$

$$= \frac{\lambda}{4\pi\epsilon_0 R}\left[2\sin\theta_0\right]$$

$\theta_0 = \tan^{-1}\left(\frac{8}{6}\right) = 53^\circ$

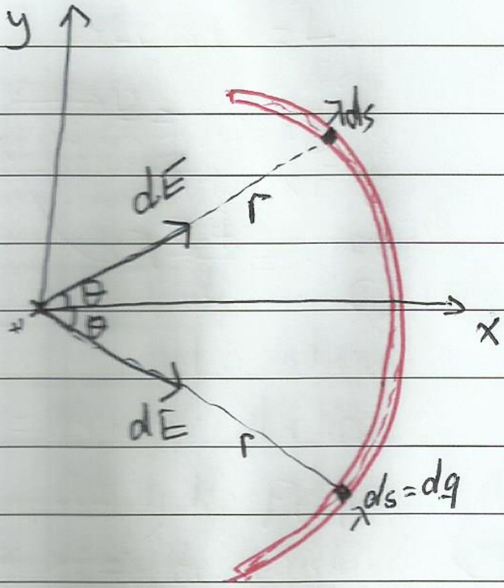
$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 R}\sin\theta_0\ \hat{j}$$

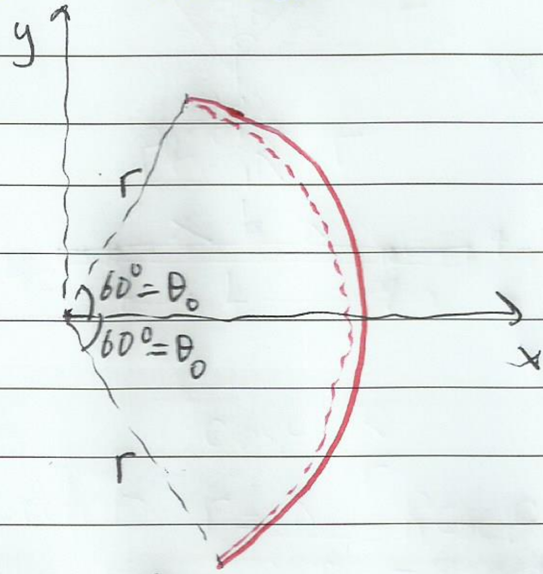
$$E = \frac{(5.78\times10^{-11})(2)\,9\times10^{9}}{6\times10^{-2}}\sin 53\ \hat{j}$$

$$\vec{E} = 13.87\hat{j}\ N/C$$

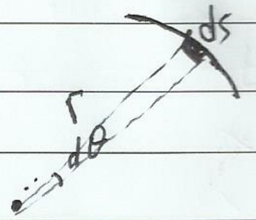
Sample Problem 22.03

Electric Field of a charged circular rod





$$dE = \frac{k\,dq}{r^2}, \quad dq = \lambda\,ds = \lambda r\,d\theta$$



$$dE = \frac{k(\lambda r\,d\theta)}{r^2}$$ from symmetry $E_y = 0$

$$(dE)_x = \frac{k\lambda r\,d\theta}{r^2}\cos\theta = \frac{k\lambda\cos\theta\,d\theta}{r}$$

$$E_x = \frac{k\lambda}{r}\int_{-60}^{+60}\cos\theta\,d\theta = \frac{k\lambda}{r}\left[\sin\theta\right]_{-60}^{+60} = \frac{k\lambda}{r}\left[\sin 60 - \sin -60\right]$$

$$E_x = \frac{\lambda}{4\pi\epsilon_0 r}(2\sin 60) = \frac{\lambda\,\sin\theta_0}{2\pi\epsilon_0 r}$$

$$E_x = \frac{1.73\lambda}{4\pi\epsilon_0 r}, \quad \lambda = \frac{Q}{\text{length}} = \frac{Q}{r(2\pi/3)} = \frac{0.477Q}{r}$$

$$= \frac{1.73(0.477Q)}{4\pi\epsilon_0 r^2} = \frac{0.83\,Q}{4\pi\epsilon_0 r^2}$$